

ASSIGNMENT: DIFFERENTIATION

- Q.1. If $y = \log \{x + \sqrt{x^2 + a^2}\}$, then prove $(x^2 + a^2)y_2 + xy_1 = 0$.
- Q.2. If $y = (x + \sqrt{1 + x^2})^n$, then show $(1 + x^2)y_2 + xy_1 - x^2y = 0$.
- Q.3. If $y = x \log \left(\frac{x}{a+bx}\right)$, then show that $x^3 \frac{d^2y}{dx^2} = \left(x \frac{dy}{dx} - y\right)^2$.
- Q.4. If $x = \tan \left(\frac{1}{a} \log y\right)$, then show that $(1 + x^2)y_2 + (2x - a)y_1 = 0$.
- Q.5. If $(x - a)^2 + (y - b)^2 = c^2$ then show $\frac{(1 + y_1^2)^{\frac{3}{2}}}{y_2} = -c$
- Q.6. If $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$, then show $\frac{dy}{dx} = \sqrt{\frac{1 - y^2}{1 - x^2}}$.
- Q.7. If $x = \sin t$; $y = \sin pt$ then show that $(1 - x^2)y_2 - xy_1 + p^2y = 0$.
- Q.8. If $y = \tan^{-1} \left(\frac{a \cos x - b \sin x}{b \cos x + a \sin x}\right)$ then show $\frac{d^2y}{dx^2} = 0$
- Q.9. If $x^{16}y^9 = (x^2 + y)^{17}$, then show that $\frac{dy}{dx} = \frac{2y}{x}$
- Q.10. Find $\frac{dy}{dx}$ for $y = \sin^{-1} \left[\frac{6x - 4\sqrt{1 - 4x^2}}{5}\right]$
- Q.11. Differentiate $\tan^{-1} \left(\frac{\sqrt{1 + x^2} - 1}{x}\right)$ w.r.t. $\sin^{-1}(2x\sqrt{1 - x^2})$.
- Q.12. If $y = a^x + x^a + x^x + a^a$ then find $\frac{dy}{dx}$.
- Q.13. If $y = \log_a x + \log_x a + \log_a a + \log_x x$ then find $\frac{dy}{dx}$.
- Q.14. If $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$, then show that $\frac{dy}{dx} = \tan \left(\frac{3\theta}{2}\right)$.
- Q.15. If $y = \tan^{-1} \left(\frac{5ax}{a^2 - 6x^2}\right)$ find $\frac{dy}{dx}$.
- Q.16. If $x^y + y^x = a^a$ find $\frac{dy}{dx}$.
- Q.17. If $y = \sin(m \sin^{-1}x)$ then show $(1 - x^2)y_2 - xy_1 + m^2y = 0$
- Q.18. Prove that $\frac{d}{dx} \left[\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right] = \sqrt{a^2 - x^2}$
- Q.19. If $y = \sin^{-1} \left[\frac{2^{x+1} \cdot 3^x}{1 + (36)^x} \right]$ find $\frac{dy}{dx}$.
- Q.20. If $y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \dots \dots \dots \infty}}}$ show $\frac{dy}{dx} = \frac{\sin x}{1 - 2y}$