Smart Mathematics
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ASSIGNMENT : VECTORS

Q1. For any vector $\vec{a}$, show that $|\vec{a}\times\hat{\imath}|^2+|\vec{a}\times\hat{\jmath}|^2+|\vec{a}\times\hat{k}|^2=2|\vec{a}|^2$

Q2. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are the vectors such that $\vec{a}.\vec{b}=\vec{a}.\vec{c}$ and $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}$ then show that $\vec{b}=\vec{c}$.

Q3. If the sum of two unit vectors is a unit vector prove that magnitude of their difference is $\sqrt{3}$.

Q4. If $\vec{a}$ and $\vec{b}$ are the unit vectors at an angle θ.

Show that :

a) $\text{Cos}\frac{\theta}{2}=\frac{1}{2}|\hat{a}+\hat{b}|$ b) $\text{Sin}\frac{\theta}{2}=\frac{1}{2}|\hat{a}-\hat{b}|$ c) $\tan\frac{\theta}{2}=\frac{|\hat{a}-\hat{b}|}{|\hat{a}+\hat{b}|}$

Q5. Prove that : a) $|\vec{a}\times\vec{b}|^2+(\vec{a}.\vec{b})^2=|\vec{a}|^2|\vec{b}|^2$

b) $|\hat{a}+\hat{b}|^2= |\hat{a}|^2+|\hat{b}|^2+2\vec{a}.\vec{b}$

Q6. Show that the Angle between two diagonals of a cube is $\text{Cos}^{-1}(\frac{1}{3})$.

Q7. If $|\vec{a}+\vec{b}|=|\vec{a}|+|\vec{b}|$ then prove $\vec{a}$ is parallel to $\vec{b}$.

Q8. Prove Cauchy Schwarty Inequality.

ie for any two vectors $\vec{a}$ and $\vec{b}$; show that $|\vec{a}.\vec{b}|\leq|\vec{a}|.|\vec{b}|$

Q9. Prove Triangle inequality i.e. for any two vectors $\vec{a}$ and $\vec{b}$

prove $|\vec{a} + \vec{b}| \leq |\vec{a}|+|\vec{b}|$.

Q10. a) If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$ then show that $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$.

b) If $\vec{a}+\vec{b}+\vec{c}=0$ then prove $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$.

c) If $|\vec{a}+\vec{b}|^2=|\vec{a}|^2+|\vec{b}|^2$ the show $\vec{a}$ and $\vec{b}$ orthogonal vectors.