MATHEMATICS

Chapter 2: RELATIONS AND FUNCTIONS

RELATIONS AND FUNCTIONS

Smart Mathematics
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Key Concepts

1. A pair of elements grouped together in a particular order is known as an ordered pair.

2. The two ordered pairs (a, b) and (c, d) are said to be equal if and only if a = c and b = d.

3. Let A and B be any two non-empty sets. The Cartesian product A × B is the set of all ordered pairs ofelements of sets from A and B defined as follows:
   $A \times B = \{(a, b) : a \in A, b \in B\}$.
   Cartesian product of two sets is also known as the product set.

4. If any of the sets of A or B or both are empty, then the set A × B will also be empty and consequently, n(A × B) = 0.

5. If the number of elements in A is m and the number of elements in set B is n, then the set A × B willhave mn elements.

6. If any of the sets A or B is infinite, then A × B is also an infinite set.

7. Cartesian product of sets can be extended to three or more sets. If A, B and C are three non-emptysets, then $A \times B \times C = \{(a, b, c): a \in A, b \in B, c \in C\}$. Here (a, b, c) is known as an ordered triplet.

8. Cartesian product of a non-empty set A with an empty set is an empty set, i.e. $A \times \Phi = \Phi$.

9. The Cartesian product is not commutative, namely A × B is not the same as B × A, unless A and B are equal.

10. The Cartesian product is associative, namely A × (B × C)=(A × B) × C

11. $R \times R = \{(a, b) : a \in R, b \in R\}$ represents the coordinates of all points in two-dimensional plane. $R \times R \times R = \{(a, b, c): a \in R, b \in R, c \in C\}$ represents the coordinates of all points in three-dimensional plane.

12. A relation R from the non-empty set A to another non-empty set B is a subset of their Cartesianproduct A × B, i.e. $R \subseteq A \times B$.

13. If $(x, y) \in R$ or x R y, then x is related to y.

14. If $(x, y) \notin R$ or $x \not{R} y$, then x is not related to y.

15. The second element b in the ordered pair (a, b) is the image of first element a and a is the pre-image of b.

16. The **Domain** of R is the set of all first elements of the ordered pairs in a relation R. In other words,domain is the set of all the inputs of the relation.

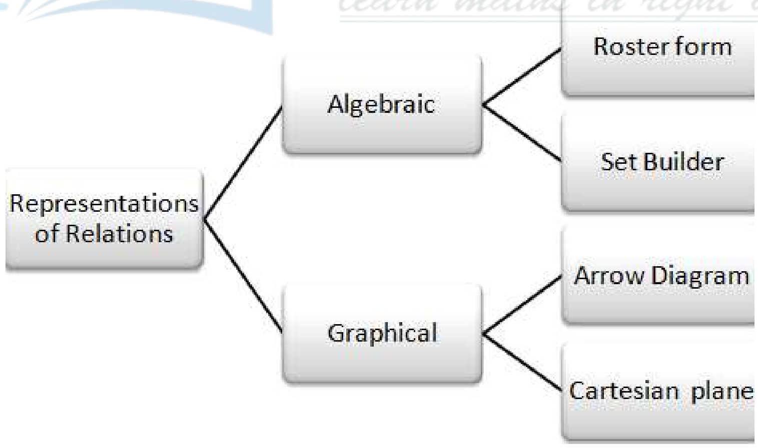
17. If the relation R is from a non-empty set A to non-empty set B, then set B is called the **co-domain** ofrelation R.

18. The set of all the images or the second element in the ordered pair (a, b) of relation R is called the **Range** of R.

19. The total number of relations that can be defined from a set A to a set B is the number is possiblesubsets of A × B.

20. A × B can have 2^{mn} subsets. This means there are 2^{mn} relations from A to B

21. Relation can be represented algebraically and graphically. The various methods are as follows



22. A relation f from a non-empty set A to another non-empty set B is said to be a function if everyelement of A has a unique image in B.

23. The domain of f is the set A. No two distinct ordered pairs in f have the same first element.

24. Every function is a relation but the converse is not true.

25. If f is a function from A to B and $(a, b) \in f$, then f (a) = b, where b is called **image** of a

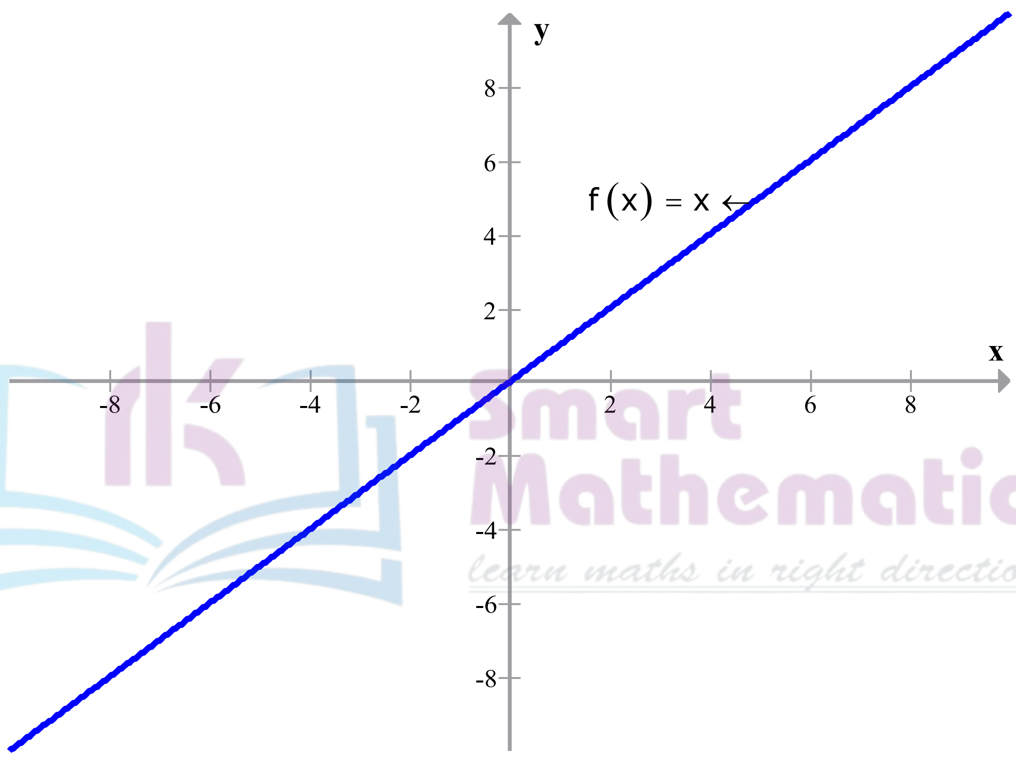
under f and a iscalled the **pre-image** of b under f.

26. If f: A $\rightarrow$ B A is the domain and B is the co domain of f.

27. The range of the function is the set of images.

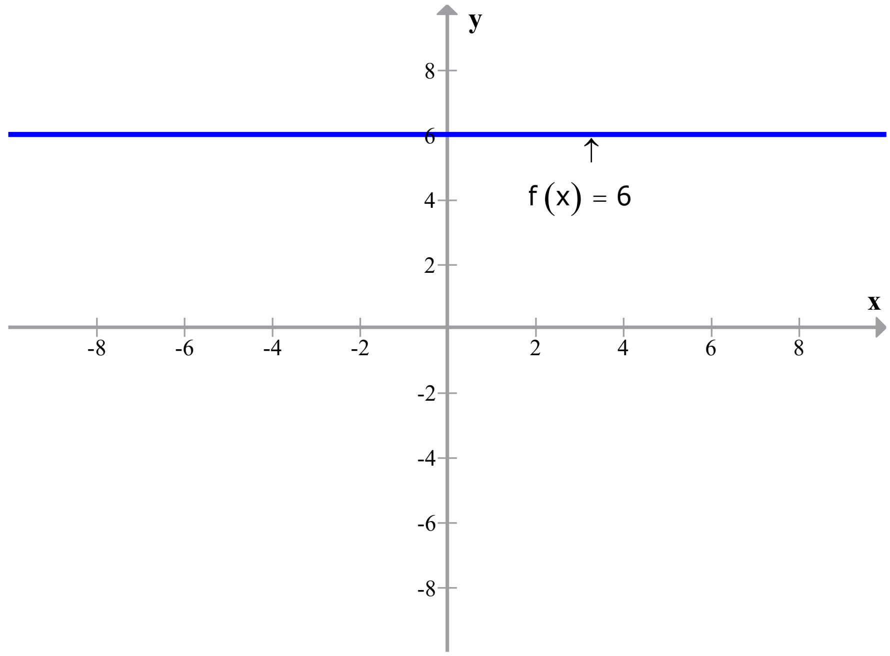
28. A real function has the set of real numbers or one of its subsets both as its domain and as its range.

29. **Identity function**: f: X $\rightarrow$ X is an identity function if f(x) = x for each x $\in$ A



30. Graph of the identity function is a straight line that makes an angle of 45° with both X- and Y–axis,respectively. All points on this line have their x and y coordinates equal.

31. **Constant function**: A constant function is one that maps each element of the domain to a constant. Domain of this function is R and range is the singleton set {c}, where c is a constant.



32. Graph of the constant function is a line parallel to the X-axis. The graph lies above X-axis if the constant c > 0, below the X-axis if the constant c < 0 and is the same as X-axis if c = 0.

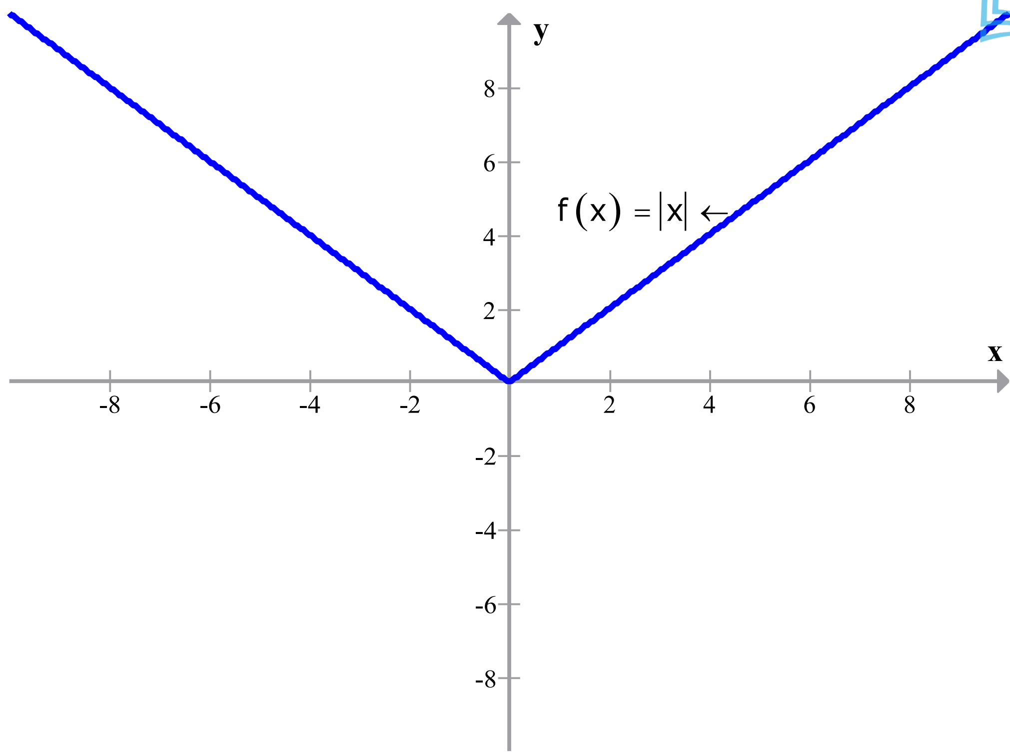
33. **Polynomial function**: f: R → R defined as $y = f(x) = a_0 + a_1x + a_2x^2 + ... + a_n x^n$, where n is a non-negative integer and $a_0, a_1, a_2, ...a_n \in R$.

34. A linear polynomial represents a straight line, while a quadratic polynomial represents a parabola.

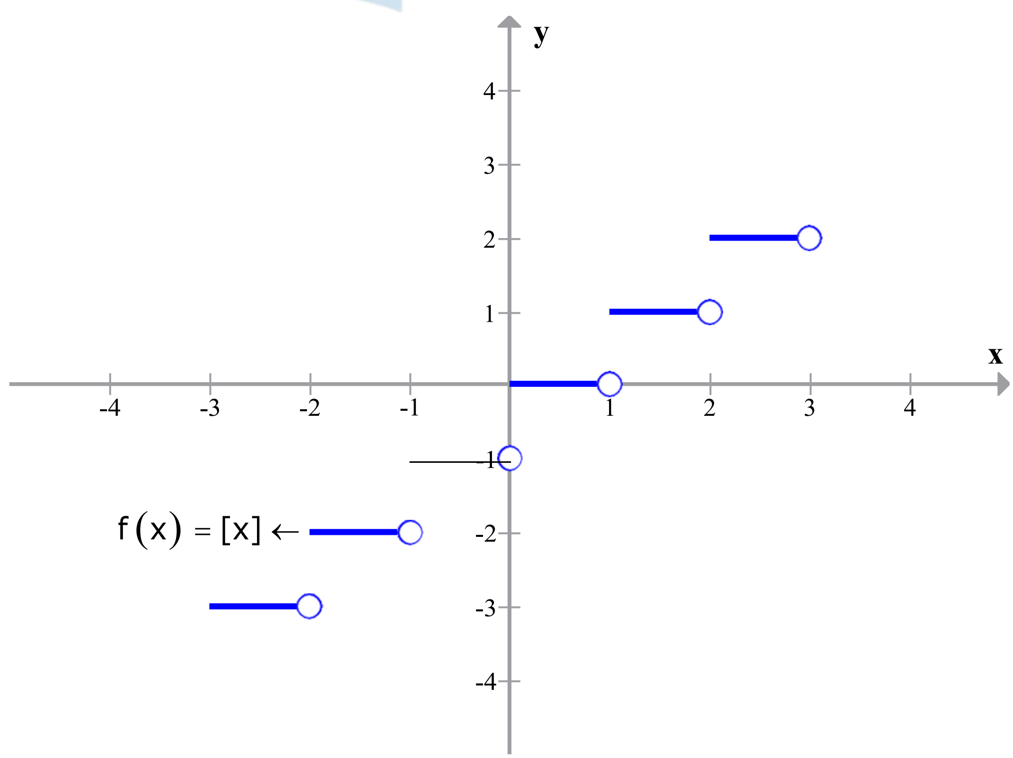
35. Functions of the form $\frac{f(x)}{g(x)}$, where f(x) and g(x) ≠ 0 are polynomial functions, are called rational functions.

36. Domain of rational functions does not include those points where g(x) = 0. For example, the domain of $f(x) = \frac{1}{x-2}$ is $R - \{2\}$.

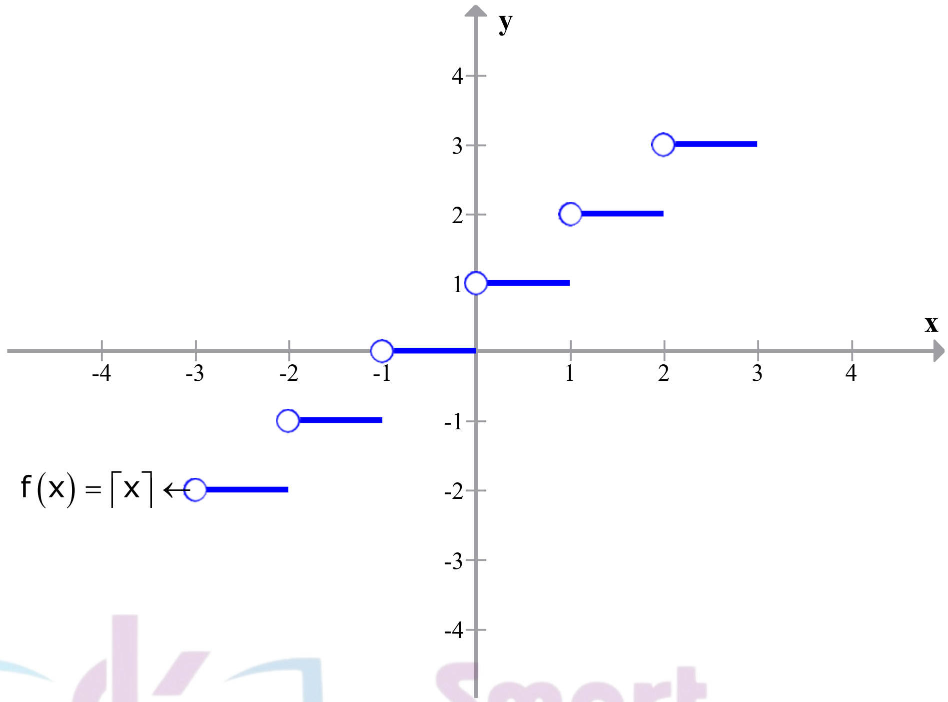
37. **Modulus function**: f: R → R defined by f(x) = |x| for each x ∈ R
f(x) = x if x ≥ 0 f(x) = −x if x<0 is called the modulus or absolute value function. The graph of modulusfunction is above the X-axis.



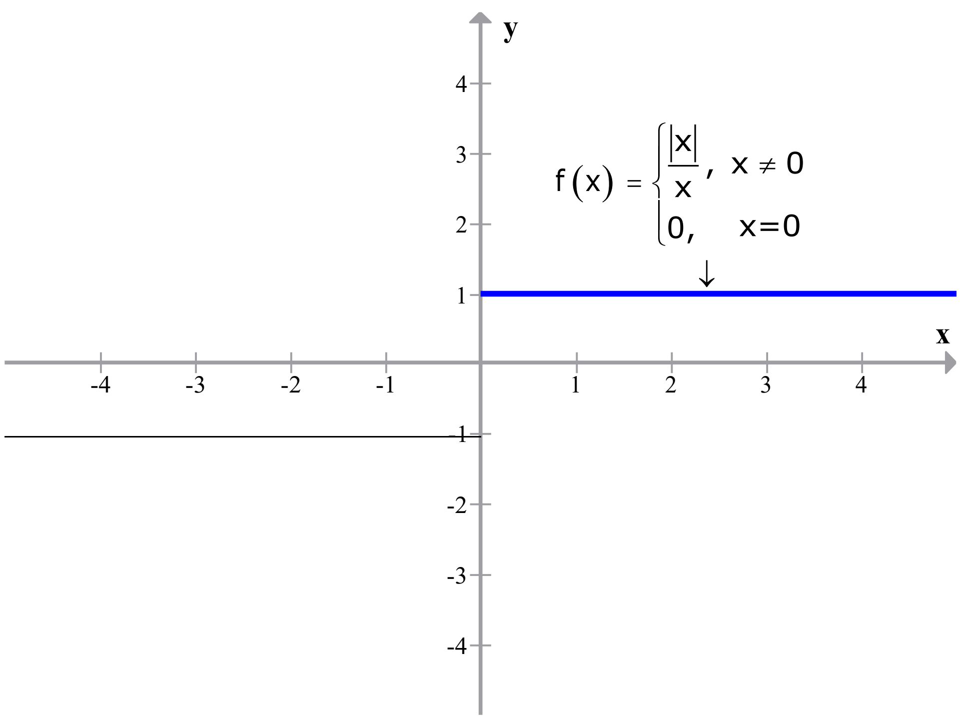
38. **Step or greatest integer function**: A function f: R → R defined by f(x) = [x], x ∈ R, where [x] is the value of greatest integer, less than or equal to x is called a step or greatest integer function. It is alsocalled as floor function.



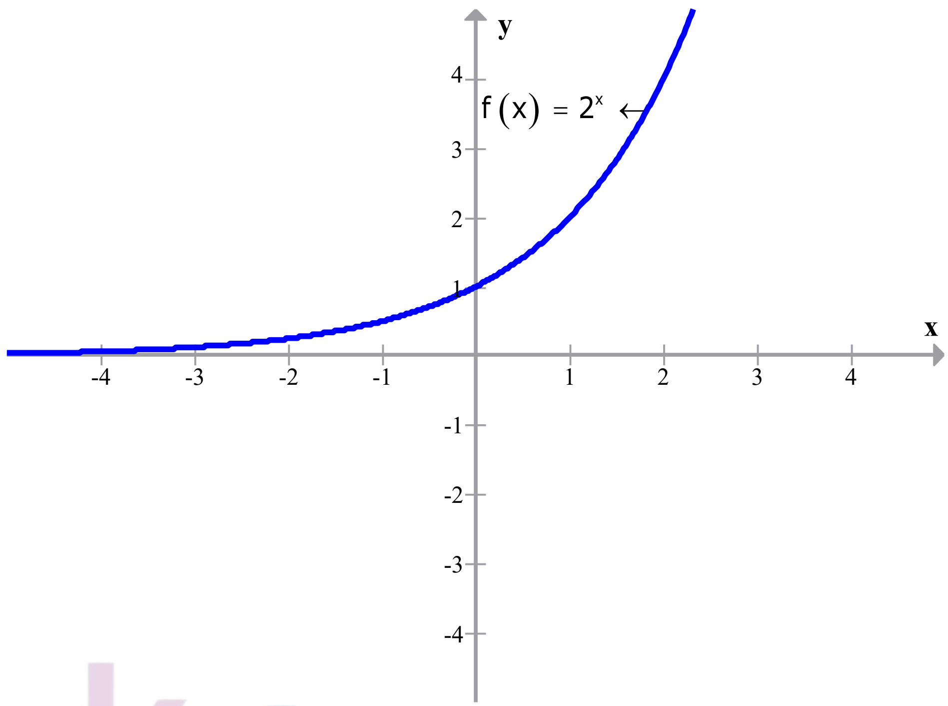
39. Smallest integer function: A function f: R $\rightarrow$ R defined by f(x) = [x], x$\in$ R where smallest integer, greater than or equal to x is called a smallest integer function. It is also known as theceiling function.



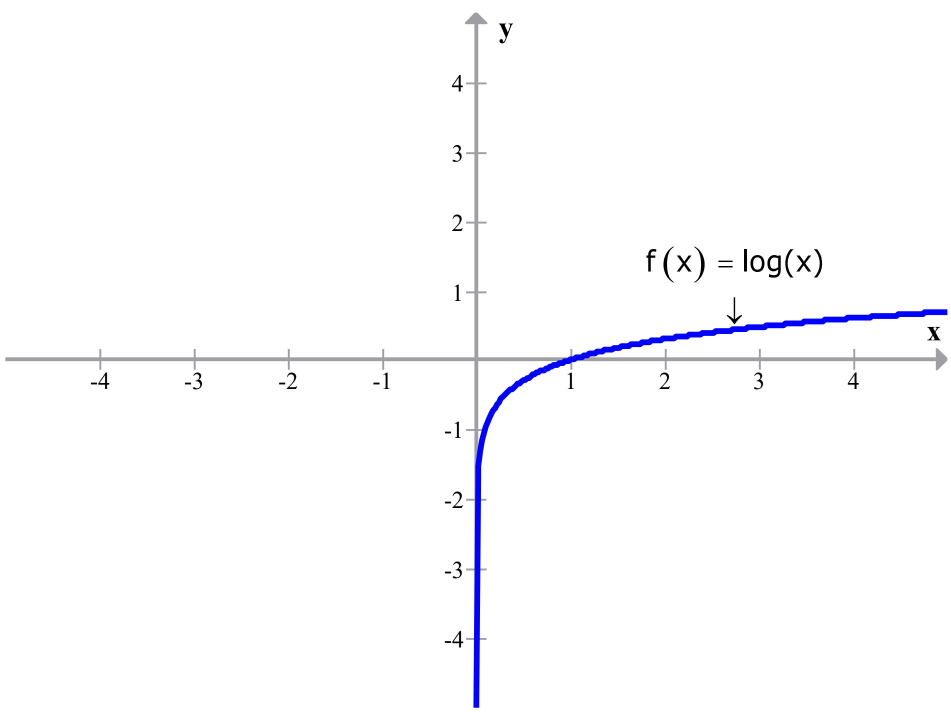
40. Signum function: $f(x) = \frac{|x|}{x}$, x $\neq$ 0 and 0 for x = 0. The domain of signum function is R and range is {−1, 0, 1}.



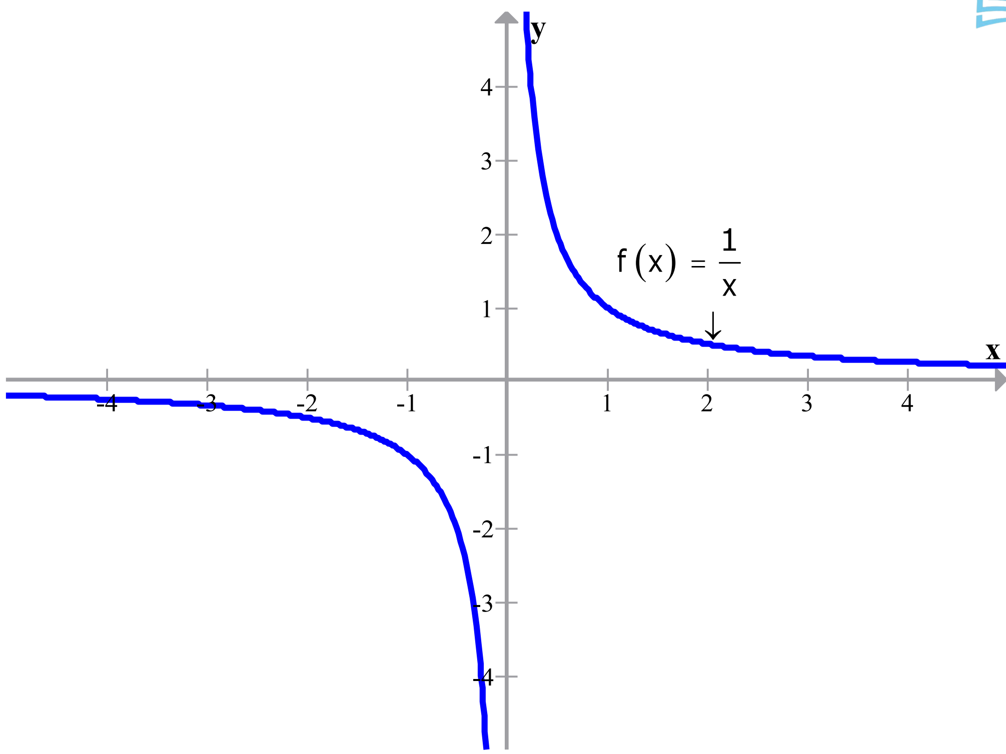
41. If a is a positive real number other than unity, then a function that relates each $x \in R$ to a^x is calledthe exponential function.



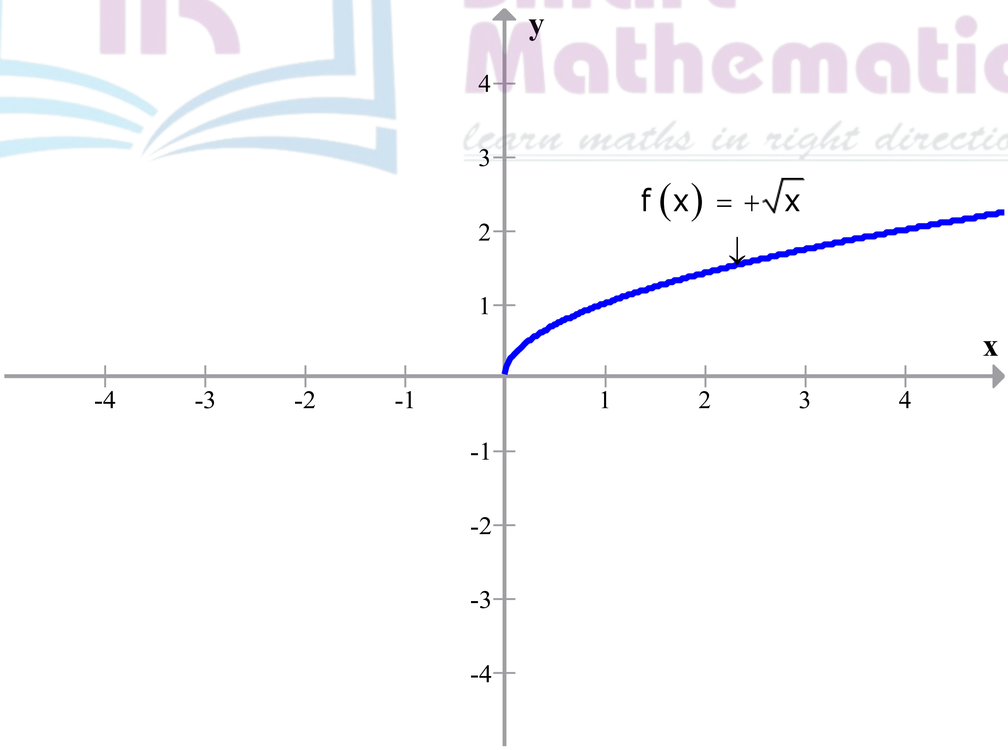
42. If $a > 0$ and $a \neq 1$, then the function defined by $f(x) = \log_a x, x > 0$ is called the logarithmicfunction.



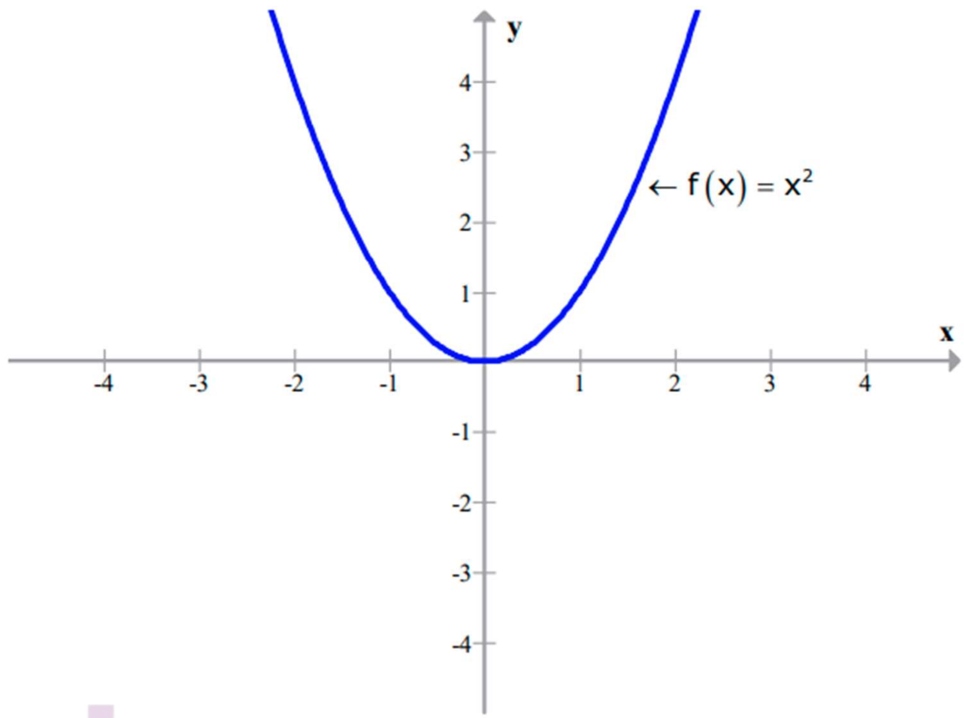
43. The function defined by $f : R - \{0\} \to R$ such that, $f(x)= \frac{1}{x}$ is called the reciprocal function.



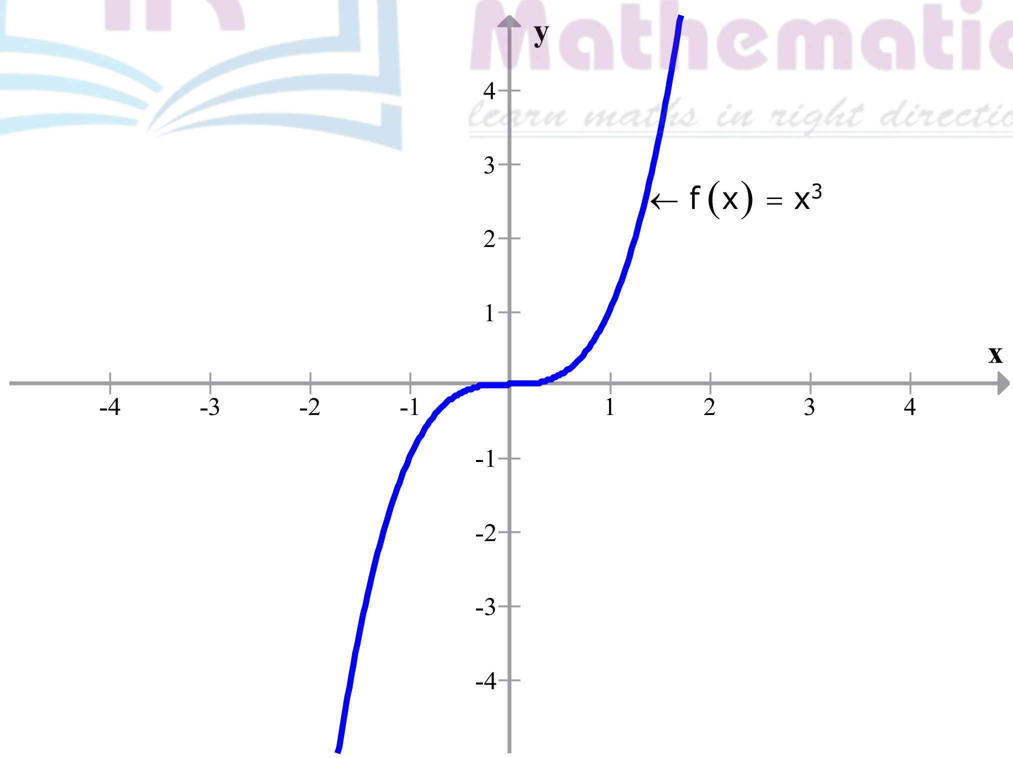
44. The function defined by $f: R^+ \to R$ such that, $f(x) = +\sqrt{x}$ is called the square root function.



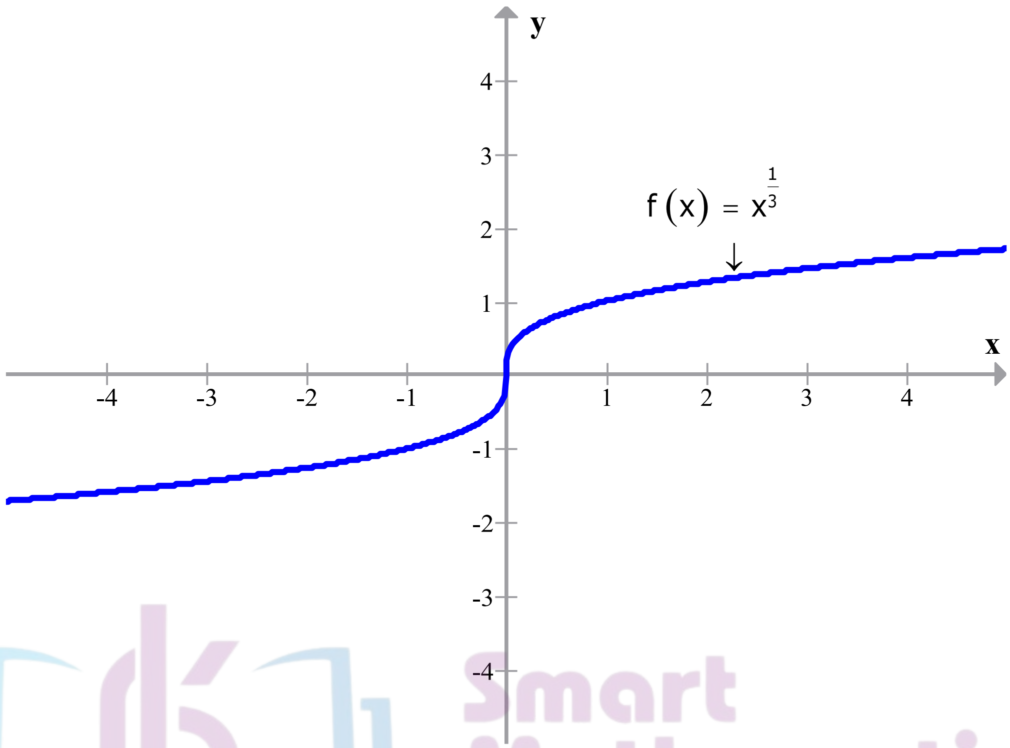
45. The function defined by $f: R \to R$ such that, $f(x) = x^2$ is called the square function.



46. The function defined by $f: R \to R$ such that $f(x) = x^3$ is called the cube function.



47. The function defined by $f: R \rightarrow R$ such that, $f(x) = x^{\frac{1}{3}}$ is called the cube root function.



Key Formulae

1. $\mathbf{R} \times \mathbf{R} = \{(x, y): x, y \in \mathbf{R}\}$
   and $\mathbf{R} \times \mathbf{R} \times \mathbf{R} = \{(x, y, z): x, y, z \in \mathbf{R}\}$

2. If (a, b) =(x, y), then a = x and b = y.

3. (a, b, c) = (d, e, f) if a = d, b = e and c = f.

4. If n (A) = n and n (B) = m, then n (A × B) = mn.

5. If n (A) = n and n (B) = m, then 2^{mn} relations can be defined from A to B.

6. **Algebra of Real function:**

   For function $f: X \rightarrow R$ and $g: X \rightarrow R$, we

   have$(f + g)(x) = f(x) + g(x), x \in X$.

$(f - g)(x) = f(x) - g(x), x \in X.$

$(f.g)(x) = f(x).\ g(x), x \in X.$

$(kf)(x) = kf(x), x \in X$, where k is a real number.

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, x \in X,\ g(x) \neq 0.$$

MIND MAP : LEARNING MADE SIMPLE CHAPTER - 2

Relations & Functions

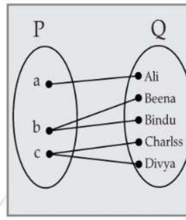
Cartesian product of sets

Given two non empty sets A&B. The cartesian product A×B is the set of all ordered pairs of elements from A&B i.e., A×B={(a,b) : a ∈ A ; b ∈ B}. If n(A)=p and n(B)=q , then n(A×B)=pq

Relation

Let A&B be two empty sets. Then any subset ' R ' of A x B is a relation from A to B.If (a,b) ∈R, then we write a R b , which is read as ' a is related to b ' by a relation R, 'b' is also called image of'a' under R. The total number of relations that can be defined from a set A to a set B is the number of possible subsets of A x B. If n(A)=p and n(B)=q, then n (A x B)=pq and total number relations is 2^{pq}.

Pictorial representation of a relation



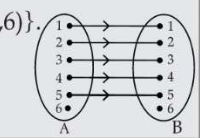
Given, R={(x,y): x is the first letter of the name y, x ∈P, y ∈Q} Then, R={(a,Ali), (b, Beena), (c, charu)}This is a visual or pictorial representation of relation R (called an arrow diagram)is shown in figure.

Domain & range of a Relation

If R is a relation from A to B, then the set of first elements in R is called domain & the set of second elements in R is called range of R. Symbolically, Domain of R={x:(x,y) ∈ R}; Range of R={y:(x,y) ∈ R} The set B is called co-domain of relation R.

Note: the range ⊆ Codomain.

Eg. Given, R={(1,2),(2,3),(3,4),(4,5),(5,6)}. then Domain of R={1,2,3,4,5} Range of R={2,3,4,5,6} and codomain of R={1,2,3,4,5,6}



Inverse relation

Let A&B be two sets and R be a relation from set A to set B.Then inverse of R ,denoted by R^{-1}, is a relation from B to Aand is defined by R^{-1}={(b,a) : (a,b) ∈ R}. Clearly, (a,b) ∈R ⇔ (b,a) ∈ R^{-1} Also, Dom(R)=Range (R^{-1}) and Range(R)=Dom(R^{-1})

Even and odd function

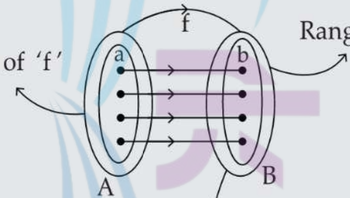
Even function $f(-x)=f(x)$, ∀x∈ Domain

Odd function $f(-x)=-f(x)$, ∀x∈ Domain

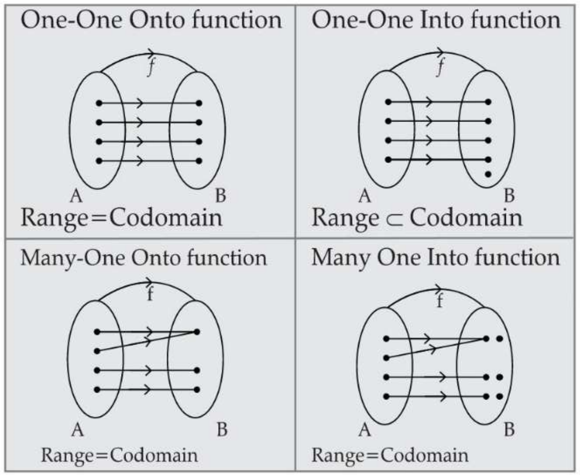
Functions

Definition: A relation ' f ' from a set A to set B is said to be a function if every element of set A has one and only one image in set B.

Notations: Domain(Input) $\xrightarrow{x}$ [f] ⟶ y=f(x) Range(Output)



Kinds of Functions



Algebra of functions

Let $f:x \to R$ and $g \to R$ be any two real functions where X⊂R.
Addition: (f+g) x = f(x)+g(x); ∀x∈ R
Subtraction: (f–g) x = f(x)–g(x); ∀ x∈ R
Product: (fg) x = f(x).g(x); ∀ x ∈ R
Quotient: (f/g) (x) = f(x)/g(x); provided g(x) ≠ 0, ∀ x ∈ R

Some standard real functions

Log function: $f(x)=\log_e x$, a>0 , a ≠ 1 Domain=$x \in (0, \infty)$ Range=y ∈ R

Identity function: The function f:R → R defined by y=f(x)=x ∀ x ∈R is called identity function. Domain=R and Range=R

Constant function: The function f:R → R defined by y=f(x)=c, ∀ x ∈ R, where c is a constant is called constant function. Domain=R and Range ={c}

Modulus function: The function f:R → R defined by $f(x)=\begin{cases} x; x \geq 0 \\ -x; x < 0 \end{cases}$ is called modulus function. It is denoted by y=f(x)=|x|. Domain=R and Range =(0, ∞)

Signum function: The function f:R → R defined by $f(x)=\begin{cases} 1, x>0 \\ 0, x=0 \\ -1, x<0 \end{cases}$ is called signum function. It is usually denoted by y=f(x)=sgn(x) Domain=R and Range ={ 0, -1, 1}

Greatest integer function: The function f:R → R defined by as the greatest integer less than or equal to x .It is usually denoted by y=f(x)=[x]. Domain=R and Range=Z(All integers)

Exponential function: $f(x)=a^x$, a>0 , a ≠ 1, Domain: x∈R: Range:f(x) ⊂ (0,∞)

Important Questions

Multiple Choice questions-

Question 1. The domain of the function ${}^{7-x}P_{x-3}$ is

(a) {1, 2, 3}

(b) {3, 4, 5, 6}

(c) {3, 4, 5}

(d) {1, 2, 3, 4, 5}

Question 2. The domain of $\tan^{-1}(2x + 1)$ is

(a) R

(b) R – {1/2}

(c) R – {-1/2}

(d) None of these

Question 3. Two functions f and g are said to be equal if f

(a) The domain of f = the domain of g

(b) The co-domain of f = the co-domain of g

(c) f(x) = g(x) for all x

(d) all of above

Question 4. If the function f : R → R be given by $f(x) = x^2 + 2$ and g : R → R is given by g(x) = x/(x – 1). The value of g of (x) is

(a) $(x^2 + 2)/(x^2 + 1)$

(b) $x^2/(x^2 + 1)$

(c) $x^2/(x^2 + 2)$

(d) None of these

Question 5. Given g(1) = 1 and g(2) = 3. If g(x) is described by the formula g(x) = ax + b, then the value of a and b is

(a) 2, 1

(b) -2, 1

(c) 2, -1

(d) -2, -1

Question 6. Let f : R → R be a function given by $f(x) = x^2 + 1$ then the value of $f^{-1}(26)$ is

(a) 5

(b) -5

(c) ±5

(d) None of these

Question 7. The function f(x) = x – [x] has period of

(a) 0

(b) 1

(c) 2

(d) 3

Question 8. The function f(x) = sin (πx/2) + cos (πx/2) is periodic with period

(a) 4

(b) 6

(c) 12

(d) 24

Question 9. The domain of the function $f(x) = x/(1 + x^2)$ is

(a) R – {1}

(b) R – {-1}

(c) R

(d) None of these

Question 10. If f: R → R is defined by $f(x) = x^2 – 3x + 2$, the f(f(y)) is

(a) $x^4 + 6x^3 + 10x^2 + 3x$

(b) $x^4 – 6x^3 + 10x^2 + 3x$

(c) $x^4 + 6x^3 + 10x^2 – 3x$

(d) $x^4 – 6x^3 + 10x^2 – 3x$

Very Short Questions:

1. Find a and b if (a – 1, b + 5) = (2, 3) If A = {1,3,5}, B = {2,3} find:

2. A × B

3. B × A

Let A = {1,2}, B = {2,3,4}, C = {4,5}, find (Question- 4,5)

4. A × (B ∩ C)

5. A × (B ∪ C)

6. If P = {1,3}, Q = {2,3,5}, find the number of relations from A to B

7. If A = {1,2,3,5} and B = {4,6,9}, R = {(x, y): |x – y| is odd, x ∈ A, y ∈ B} Write R in roster form Which of the following relations are functions? Give reason.

8. R = {(1,1), (2,2), (3,3), (4,4), (4,5)}

9. R = {(2,1), (2,2), (2,3), (2,4)}

10. R = {(1,2), (2,5), (3,8), (4,10), (5,12), (6,12)} Which of the following arrow diagrams represent a function? Why?

Short Questions:

1. Let A = {1,2,3,4}, B = {1,4,9,16,25} and R be a relation defined from A to B as, R = {(x, y) : x ϵ A, y ϵ B and y = x2}

   (a) Depict this relation using arrow diagram.

   (b) Find domain of R.

   (c) Find range of R.

   (d) Write co-domain of R.

2. Let R = { (x, y) : x, y ϵ N and y = 2x} be a relation on N. Find :

   (i) Domain

   (ii) Codomain

   iii) Range

   Is this relation a function from N to N

3. Find the domain and range of, f(x) = |2x – 3| – 3

4. Draw the graph of the Constant function, f : R ϵ R; f(x) = 2 ∀ x ϵ R. Also find its domain and range.

5. Let R = {(x, - y) : x, y ϵ W , 2x + y = 8) then

   (i) Find the domain and the range of R (ii) Write R as a set of ordered pairs.

6. Let R be a relation from Q to Q defined by R = {(a,b) : a, b ϵ Q and a -b ϵ z), Show that.

   $(i)(a,a)\in R$ for all $a\in Q$ $(ii)(a,b)\in R$ implies that $(b,a)\in R$

   $(iii)(a,b)\in R$ and $(b,c)\in R$ implies that $(a,c)\in R$

7. If $f(x)=\frac{x^2-3x+1}{x-1}$, find $f(-2)+f\left(\frac{1}{3}\right)+$

8. Find the domain and the range of the function f(x) = $3x^2 – 5$. Also find f (-3) and the

numbers which are associated with the number 43 m its range.

9. If f (x) = x^2 - 3x + 1, find x such that f (2x) = 2 f(x).

10. Find the domain and the range of the function $f(x) - \sqrt{x-1}$.

Long Questions:

1. Draw the graphs of the following real functions and hence find their range

$f(x) = \frac{1}{x}, x \in R, x \neq 0$

2. If f(x) = $x - \frac{1}{4}$, Prove that $[f(x)]^3 = f(x^3) + 3f\left(\frac{1}{x}\right)$

3. Draw the graphs of the following real functions and hence find their range

4. Let f be a function defined by $F: x \rightarrow 5x^2 + 2, x \in R$

   (i) find the image of 3 under f.

   (ii) find f(3) + f(2).

   (iii) find x such that f(x) = 22

5. The function $f(x) = \frac{9x}{5} + 32$ is the formula to connect x˚c to Fahrenheit units find (i) f(0) (ii) f(-10) (iii) the value of x f(x) = 212 interpret the result is each case.

Assertion Reason Questions:

1. In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as.

   Assertion (A) : If (x +1, y – 2) = (3, 1), then x = 2 and y = 3.

   Reason (R) : Two ordered pairs are equal if their corresponding elements are equal.

   (i) Both assertion and reason are true and reason is the correct explanation of assertion.

   (ii) Both assertion and reason are true but reason is not the correct explanation of assertion.

   (iii) Assertion is true but reason is false.

   (iv) Assertion is false but reason is true.

2. In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as.

   Assertion (A) : The cartesian product of two non-empty sets P and Q is denoted as P × Q and P × Q = {(p, q) : p ∈ P, q ∈ Q}.

   Reason (R) : If A = {red, blue} and B = {b, c, s}, then A × B = {(red, b), (red, c), (red), (blue, b) (blue, c) (blue, s)}

(i) Both assertion and reason are true and reason is the correct explanation of assertion.

(ii) Both assertion and reason are true but reason is not the correct explanation of assertion.

(iii) Assertion is true but reason is false.

(iv) Assertion is false but reason is true.

Answer Key:

MCQ

1. (c) {3, 4, 5}

2. (a) R

3. (d) all of above

4. (a) $(x^2 + 2)/(x^2 + 1)$

5. (c) 2, -1

6. (c) ±5

7. (b) 1

8. (a) 4

9. (c) R

10. (d) $x^4 - 6x^3 + 10x^2 - 3x$

Very Short Answer:

1. a = 3, b = –2

2. A × B = {(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)

3. B × A = { (2,1), (2,3), (2,5), (3,1), (3,3), (3,5)

4. {(1,4), (2,4)

5. {(1,2), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (2,5)

6. $2^6 = 6$

7. R = { (1,4), (1,6), (2,9), (3,4), (3,6), (5,4), (5,6)

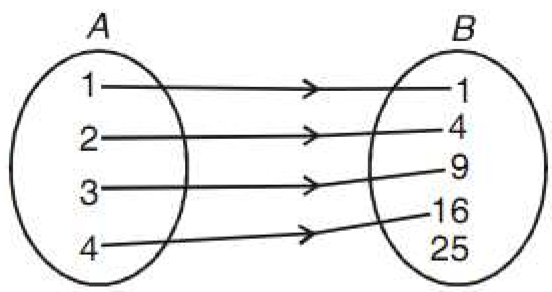
8. Not a function because 4 has two images.

9. Not a function because 2 does not have a unique image.

10. Function

Short Answer:

1.

(a) 

(b) {1,2,3,4}

(c) {1,4,9,16}

(d) {1,4,9,16,25}

2. (i) N

(ii) N

(iii) Set of even natural numbers

yes, R is a function from N to N.

3. Domain is R

Range is [–3, ∞)

4. Domain = R

Range = {2}

5. (i) Given and 2x + y = 8 and $x, y \in w$

Put

$x = 0, 2\times0 + y = 8 \Rightarrow y = 8,$

$x = 1, 2\times1 + y = 8 \Rightarrow y = 6,$

$x = 2, 2\times2 + y = 8 \Rightarrow y = 4,$

$x = 3, 2\times3 + y = 8 \Rightarrow y = 2,$

$x = 4, 2\times4 + y = 8 \Rightarrow y = 0$

for all other values of $x, y \in w$ we do not get $y \in w$

∴ Domain of R = {0,1 2, 3, 4} and range of R = {8, 6, 4, 2, 0}

(ii) R as a set of ordered pairs can be written as

$R = \{(0,8),(1,6),(2,4),(3,2),(4,0)\}$

6.

$R = [(a,b) : a, b \in Q \text{ and } a - b \in z]$

(i) For all $a \in Q, a-a=0$ and $0 \in z$, it implies that $(a,a) \in R$.

(ii) Given $(a,b) \in R \Rightarrow a-b \in z \Rightarrow -(a-b) \in z$

$\Rightarrow b-a \in z \Rightarrow (b,a) \in R.$

(iii) Given $(a,b) \in R$ and $(b,c) \in R \Rightarrow a-b \in z$ and $b-c \in z \Rightarrow (a-b)+(b-c) \in z$

$\Rightarrow a-c \in z \Rightarrow (a-c) \in R.$

7.

Given $f(x)=\dfrac{x^2-3x+1}{x-1}, Df=R-\{1\}$

$$\therefore f(-2)=\frac{(-2)^2-3(-2)+1}{-2-1}-\frac{4+6+1}{-3}=1\frac{1}{3} \text{ and}$$

$$f\left(\frac{1}{3}\right)=\frac{\left(\frac{1}{3}\right)^2-3\times\frac{1}{3}+1}{\frac{1}{3}-1}=\frac{\frac{1}{9-1+1}}{-\frac{2}{3}}=\frac{\frac{1}{9}}{-\frac{2}{3}}=\frac{1}{9}\times\left(-\frac{3}{2}\right)=-\frac{1}{6}$$

$$\therefore f(-2)+f\left(\frac{1}{3}\right)=-\frac{11}{3}-\frac{1}{6}=\frac{-22-1}{6}=\frac{-23}{6}=3\frac{5}{6}.$$

8.

Given $f(x)=3x^2-5$

For $Df, f(x)$ must be real number

$\Rightarrow 3x^2-5$ must be a real number

Which is a real number for every $x \in R$

$\Rightarrow Df = R$..........(i)

$for\ Rf$, let $y = f(x) = 3x^2 - 5$

We know that for all $x \in R, x^2 \geq 0 \Rightarrow 3x^2 \geq 0$

$\Rightarrow 3x^2 - 5 \geq -5 \Rightarrow y \geq -5 \Rightarrow Rf = [-5, \infty]$

Funthes, as $-3 \in Df$, $f(-3)$ exists is and $f(-3)$

$= 3(-3)^2 - 5 = 22.$

As $43 \in Rf$ on putting $y = 43$ is (i) weget

$3x^2 - 5 = 43 \Rightarrow 3x^2 = 48 \Rightarrow x^2 = 16 \Rightarrow x = -4, 4.$

There fore -4 and 4 are number

(is Df) which are associated with the number 43 in Rf

9.

Given $f(x) = x^2 - 3x + 1, Df = R$

$\therefore f(2x) = (2x)^2 - 3(2x) + 1 = 4x^2 - 6x + 1$

As $f(2x) = f(x)$ (Given)

$\Rightarrow 4x^2 - 6x + 1 = x^2 - 3x + 1$

$\Rightarrow 3x^2 - 3x = 0 \Rightarrow x^2 - x = 0 \Rightarrow x(x-1) = 0$

$\Rightarrow x = 0, 1.$

10.

Given $f(x) = \sqrt{x-1},$

for Df, $f(x)$ must be a real number

$\Rightarrow \sqrt{x-1}$ must be a real number

$\Rightarrow x - 1 \geq 0 \Rightarrow x \geq 1$

$\Rightarrow Df = [1, \infty]$

for Rf, let $y = f(x) = \sqrt{x-1}$

$\Rightarrow \sqrt{x-1} \geq 0 \Rightarrow y \geq 0$

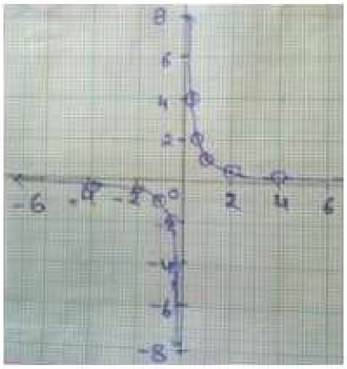
$\Rightarrow Rf = [0, \infty]$

Long Answer:

1.

Given $f(x)=\frac{1}{x}, x\in R, x\neq 0$

Let $y=f(x)=i\ell y=\frac{1}{x}, x\in R, x\neq 0$



(Fig for Answer 11)

x	-4	-2	-1	-0.5	-0.25	0.5	1	2	4
$y=\frac{1}{x}$	-0.25	-0.5	-1	-2	-4	2	1	0.5	0.25

Plot the points shown is the above table and join there points by a free hand drawing.

Portion of the graph are shown the right margin

From the graph, it is clear that Rf = R – [0]

This function is called reciprocal function.

2.

If $f(x)=x-\frac{1}{x}$, prove that $[f(x)]^3=f(x^3)+f\left(\frac{1}{x}\right)$

Given $f(x)=x-\frac{1}{x}, Df=R-[0]$

$\Rightarrow f(x^3)=x^3-\frac{1}{x^3}$ and $f\left(\frac{1}{x}\right)=\frac{1}{x}-\frac{1}{\frac{1}{x}}=\frac{1}{x}-x.........(i)$

$\therefore [f(x)]^3=\left(x-\frac{1}{x}\right)^3=x^3-\frac{1}{x^3}-3x.\frac{1}{x}\left(x-\frac{1}{x}\right)$

$$= x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$$

$$= x^3 - \frac{1}{x^3} + 3\left(\frac{1}{x} - x\right)$$

$$= f\left(x^3\right) + 3f\left(\frac{1}{x}\right) \left[\text{using } (i)\right]$$

3.

(i)Given, f(x) i.e. y = x - 1 which is first degree equation in x, y and hence it represents a straight line. Two points are sufficient to determine straight lint uniquely

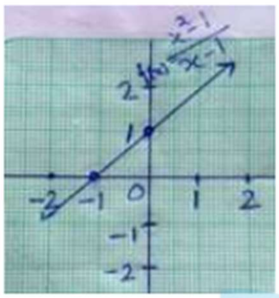


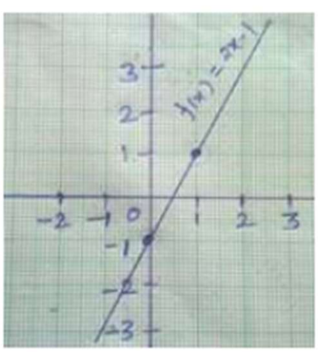
Table of values

x	**0**	**1**
y	-1	1

A portion of the graph is shown in the figure from the graph, it is clear that y takes all real values. It therefore that

$R_F = R$

(ii) Given $f(x) = \frac{x^2-1}{x-1} \Rightarrow D_F = R - (1)$



Let $y = f(x) = \frac{x^2-1}{x-1} = x + 1 (\because x \neq 1)$

i.e y = x + 1 which is a first degree equation is and hence it represents a straight line. Two points are sufficient to determine a straight line uniquely

Table of values

x	-1	0
y	0	1

A portion of the graph is shown is the figure from the graph it is clear that y takes all real values except 2. It fallows that R_F = R – [2].

4.

Given $f(x) = 5x^2 + 2, x \in R$

(i) $f(3) = 5 \times 3^2 + 2 = 5 \times 9 + 2 = 47$

(ii) $f(2) = 5 \times 2^2 + 2 = 5 \times 4 + 2 = 22$

$\therefore f(3) \times f(2) = 47 \times 22 = 1034$

(iii) $f(x) = 22$

$\Rightarrow 5x^2 + 2 = 22$

$\Rightarrow 5x^2 = 20$

$\Rightarrow x^2 = 4$

$\Rightarrow x = 2, \ -2$

5.

$f(x) = \frac{9x}{5} + 32(given)$

$(i)\ f(0) = \left(\frac{9 \times 0}{5} + 32\right) = 32 \Rightarrow f(0) = 32 \Rightarrow 0^\circ c = 32^\circ F$

$(ii)\ f(-10) = \left(\frac{9 \times (-10)}{5} + 32\right) = 14 \Rightarrow f(-10) = 14^\circ \Rightarrow (-10)^\circ c = 14^\circ F$

$(iii)\ f(x) = 212 \Leftrightarrow \frac{9x}{5} + 32 = 212 \Leftrightarrow 9x = 5 \times (180)$

$\Leftrightarrow x = 100$

$\therefore 212^\circ f = 100^\circ c$

Assertion Reason Answer:

1. (i) Both assertion and reason are true and reason is the correct explanation of assertion.
2. (i) Both assertion and reason are true and reason is the correct explanation of assertion.