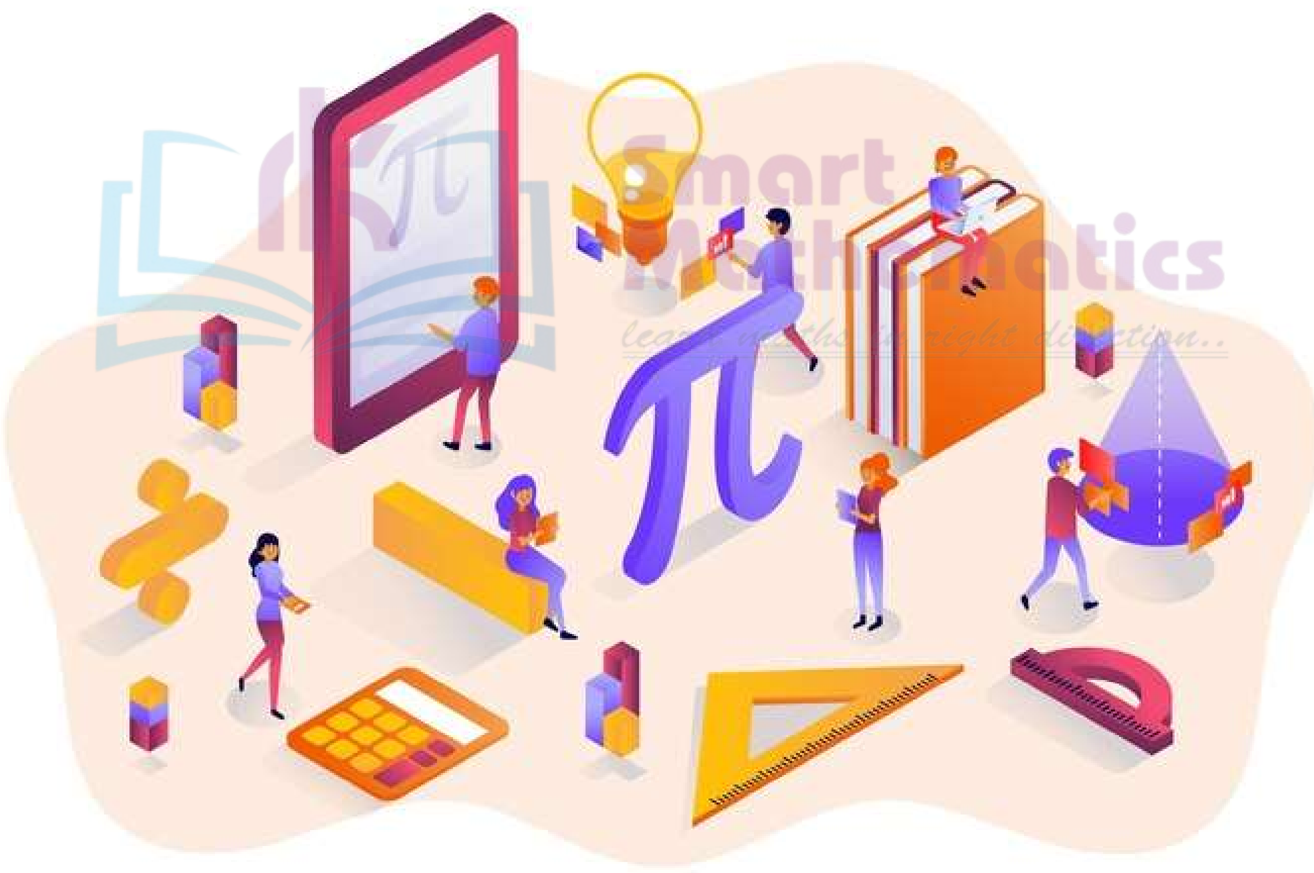
# MATHEMATICS

## Chapter 7: PERMUTATION AND COMBINATION



# PERMUTATION AND COMBINATION

**Key Concepts**

1. **Fundamental principles of counting:** There are two fundamental principles of counting:
   (i) Multiplication principle
   (ii) Addition principle

2. **Multiplication principle:** If an event can occur in **M** different ways following which another event canoccur in **N** different ways, then the total number of occurrences of the events in the given order is **M × N**. This principle can be extended to any number of finite events. Keyword here is ‘And’.

3. **Addition principle:** If there are two jobs such that they can be performed independently in **M** and **N** ways, respectively, then either of the two jobs can be performed in **M** + **N** ways. This principle can beextended to any number of finite events. Keyword here is ‘OR’.

4. The notation ‘n!’ represents the product of the first n natural numbers.

   n! = 1.2.3.4.......n

5. A permutation is an arrangement in a definite order of a number of objects taken some or all at a time.In permutations, order is important.

6. The number of permutation of n different objects taken r at a time, where $0 < r \leq n$ and the objects do not repeat is $n(n-1)(n-2)...(n-r+1)$ denoted by $^{n}P_{r}$.

7. The number of permutation of n different objects taken r at a time, where repetition is allowed is $^{n}P$.

8. The number of permutation of n objects taken all at a time, where p objects are of one kind and q are of another kind, such that p + q = n is given by $\frac{n!}{p!q!}$.

9. The number of permutation of n objects, where $p_1$ are of one kind, $p_2$ are of second kind… $p_k$ are of $k^{th}$ kind, such that $p_1 + p_2 + \ldots + p_k = n$ is $\frac{n!}{p_1!p_2!\ldots p_k!}$.

10. The number of permutation of n objects, where p objects are of one kind, q are of another kind and remaining are all distinct is given by $\frac{n!}{p!q!}$.

11. Assume that there are k things to be arranged with repetitions. Let $p_1, p_2, p_3, \ldots p_k$ be integers such that the first object occurs exactly $p_1$ times, the second occurs exactly $p_2$ times and so on. Then the total number of permutations of these k objects with the above condition is $\frac{(p_1+p_2+p_3+\ldots+p_k)!}{p_1!p_2!\ldots p_k!}$.

12. Keyword of permutations is ‘arrangement’.

13. Combination is a way of selecting their objects from a group, irrespective of their arrangements.

14. Permutations and Combinations:

| **Permutations** | **Combinations** |
|---|---|
| Arrangement in a definiteorder is considered. | Selection is made irrespective of the arrangement. |
| Ordering of the objects is essential. | Ordering of the selectedobject is immaterial. |
| Permutation corresponds to onlyone combination. | Combination corresponds tomany permutations. |
| Number of permutations exceedsthe number of combinations. | Number of combinations islesser than the number of permutations. |

15. The number of combinations or selection of r different objects out of *n* given different objects is $^nC_r$ andis given by

$^nC_r = \frac{n!}{r!(n-r)!}\ 0 \le r \le n$

16. Number of combinations of n different things taking nothing at all is considered to be 1.

17. Counting combinations is merely counting the number of ways in which some or all objects at a timeare selected.

18. The keyword for combinations is 'selection'.

19. Selecting r objects out of n objects is the same as rejecting n – r objects, so $^nC_{n-r} = {^nC_r}$.

**Key Formulae**

1. n! = 1 × 2 × 3 × …× n or n! = n × (n – 1)!
2. n! is defined for positive integers only.
3. n! = n(n – 1)(n – 2)!(provided n ≥ 2)
4. n! = n n(n – 1)(n – 2)(n – 3)! (provided n ≥ 3)
5. 0! = 1! = 1
6. $^nP_r = \frac{n!}{(n-r)!},\ 0 \le r \le n$
7. $^nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!$
8. $^nP_0 = \frac{n!}{(n-0)!} = \frac{n!}{n!} = 1$
9. If $P_m$ represents $^mP_m$, then $1 + 1 \cdot P_1 + 2 \cdot P_2 + 3 \cdot P_3 + \ldots + n \cdot P_n = (n+1)!$
10. $^nC_r = \frac{n!}{r!(n-r)!}\ 0 \le r \le n$
11. $^nC_x = {^nC_y} \Leftrightarrow x + y = n$ or x=y

12. Pascal's rule: If n and k are non-negative integers such that $k \le n$, then $^nC_k + {}^nC_{k-1} = {}^{n+1}C_k$.

13. If n and k re non-negative integers such that $1 \le k \le n$, then $^nC_k = \frac{n}{k} \times {}^{n-1}C_{k-1}$

14. If n and k are non-negative integers such that $1 \le k \le n$, then $n \times {}^{n-1}C_{k-1} = (n-k+1) \times {}^nC_{k-1}$

15. The greatest among $^nC_0, {}^nC_1, {}^nC_2, \ldots {}^nC_n$ is $^nC_{\frac{n}{2}}$ when n is an even natural number.

16. The greatest among $^nC_0, {}^nC_1, {}^nC_2, \ldots {}^nC_n$ is $^nC_{\frac{n-1}{2}}$ or $^nC_{\frac{n+1}{2}}$ when n is an odd natural number.

17. $^nP_r = {}^nC_r \times r!,\ 0 < r \le n$

18. $^nC_0 = 1$

19. $^nC_0 = {}^nC = 1$

20. $^nC_n \quad {}^nC_1 = n$

21. $^nC_{n-r} = \frac{n!}{(n-r)!(n-(n-r))!} = \frac{n!}{(n-r)!r!} = {}^nC_r$

# MIND MAP : LEARNING MADE SIMPLE CHAPTER - 7

## Permutations & Combinations

### Combinations

Each of different selections made by taking source or all of a number of distinct objects or item, irrespective of their arrangements or order in which they are placed, is called a combination.

The no. of combinations of $n$ different things taken $r$ at a time, denoted by ${}^nC_r$ is given by

$$ {}^nC_r = \frac{n!}{r!(n-r)!};\ 0 \le r \le n $$

### Fundamental Principle of Counting (F.P.C.)

**F.P.C. of Multiplication:** If an event can occur in $m$ different ways, following which another event in $n$ different ways, then the total number of occurance of the events in the given order is $m \times n$

**F.P.C. of Addition:** If there are two events such that they can performed independently in $m$ and $n$ ways respectively, then either of the two events can be performed in $(m+n)$ ways.

### Permutation

Each of the different arrangements which can be made by taking some or all of a number of distinct objects is called a **permutation**.

The no. of permutations of $n$ different things taken $r$ at a time, where repetition is not allowed, is denoted by ${}^nP_r$ and is given by

$$ {}^nP_r = \frac{n!}{(n-r)!} $$

### Permutations under given Conditions

The no. of all permutations of 'n' different objects taken r at a time:

(i) When a particular object is to be always included in each arrangement is ${}^{n-1}P_r$

(ii) When a particular object is never taken in each arrangement in ${}^{n-1}P_r$

(iii) The no. of permutation of $n$ different things taken r at a time, where repetition is allowed is $(n)^r$

### Permutations of alike Objects

The no. of permutations of n objects taken all at a time, where $p_1$ objects are of first kind, $p_2$ objects are of the second kind, ....$p_k$ objects are of the $k^{th}$ kind and rest, if any one are all different is $\frac{n!}{p_1!p_2!p_3!\dots p_k!}$

### Circular Permutations

1. The no. of circular permutations of '$n$' distinct objects is $(n-1)!$
2. If anti-clockwise and clockwise order of arrangements are not distinct then the no. of circular permutations of $n$ distinct items is $\frac{1}{2}(n-1)!$

**e.g.:** arrangements of beads in a necklace, arrangements of flower in a garland etc.

### Some Special Results of ${}^nC_r$

Since,

$$ {}^nC_r = \frac{n!}{r!(n-r)!} $$

- In particular, $r = n$
- ${}^nC_r = \frac{n!}{n!0!} = 1$
- ${}^nC_{n-r} = {}^nC_r$
- ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

### Relation between ${}^nP_r$ and ${}^nC_r$

${}^nP_r = {}^nC_r \times r!$ $\quad \because 0 < r \le n$

*i.e.*, Corresponding to each combination of ${}^nC_r$, we have r! permutations, because $r$ objects in every combination can be rearranged in $r!$ ways

### Combinations under Certain Conditions

- The no. of arrangements of n distinct objects taken r at a time, so that k particular objects are
  (i) Always included: ${}^{n-k}C_{r-k}.r!$
  (ii) Never included: ${}^{n-k}C_r.r!$
- The no. of selections from '$n$' different objects, taking at least one $= {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots\dots\dots {}^nC_n = 2^n - 1$

# Important Questions

## Multiple Choice questions-

Question 1. It is required to seat 5 men and 4 women in a row so that the women occupy the even places. The number of ways such arrangements are possible are

(a) 8820

(b) 2880

(c) 2088

(d) 2808

Question 2. Six boys and six girls sit along a line alternately in x ways and along a circle (again alternatively in y ways), then

(a) $x = y$

(b) $y = 12x$

(c) $x = 10y$

(d) $x = 12y$

Question 3. How many 3-letter words with or without meaning, can be formed out of the letters of the word, LOGARITHMS, if repetition of letters is not allowed

(a) 720

(b) 420

(c) none of these

(d) 5040

Question 4. A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of at least 3 girls

(a) 588

(b) 885

(c) 858

(d) None of these

Question 5. In how many ways can 12 people be divided into 3 groups where 4 persons must be there in each group?

(a) none of these

(b) $12!/(4!)^3$

(c) Insufficient data

(d) $12!/\{3! \times (4!)^3\}$

Question 6. How many factors are $2^5 \times 3^6 \times 5^2$ are perfect squares

(a) 24

(b) 12

(c) 16

(d) 22

Question 7. If ${}^nC_{15} = {}^nC_6$ then the value of ${}^nC_{21}$ is

(a) 0

(b) 1

(c) 21

(d) None of these

Question 8. If ${}^{n+1}C_3 = 2\ {}^nC_2$, then the value of n is

(a) 3

(b) 4

(c) 5

(d) 6

Question 9. There are 15 points in a plane, no two of which are in a straight line except 4, all of which are in a straight line. The number of triangle that can be formed by using these 15 points is

(a) 15C3

(b) 490

(c) 451

(d) 415

Question 10. In how many ways in which 8 students can be sated in a circle is

(a) 40302

(b) 40320

(c) 5040

(d) 50040

## Very Short:

**1.** Evaluate 4! – 3!

2. If $^nc_a = {}^nc_b$ find n
3. The value of 0! Is?
4. Given 5 flags of different colours here many different signals can be generated if each signal requires the use of 2 flags. One below the other
5. How many 4 letter code can be formed using the first 10 letter of the English alphabet, if no letter can be repeated?
6. A coin is tossed 3 times and the outcomes are recorded. How many possible out comes are there?
7. Compute $\frac{8!}{6!\times 2!}$
8. If $^nC_3 = {}^nC_2$ find $^nC_2$
9. In how many ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colours.
10. Find r, if $5.4\ P_r = 6.5P_{r-1}$

## Short Questions:

1. How many words, with or without meaning can be made from the letters of the word MONDAY. Assuming that no. letter is repeated, it

   (i) 4 letters are used at a time

   (ii) All letters are used but first letter is a vowel?
2. Prove that:

   $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
3. A bag contains 5 black and 6 red balls determine the number of ways in which 2 black and 3 red balls can be selected.
4. In how many ways can 5 girls and 3 boys be seated in a row so that no two boys are together?
5. How many words, with or without meaning, each of 3 vowels and 2 consonants can be formed from the letters of the word INVOLUTE.

## Long Questions:

1. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has :

   (i) no girl? (ii) at least one boy and one girl? (iii) at least 3 girls?
2. Find the number of words with or without meaning which can be made using all the letters of the word. AGAIN. If these words are writer as in a dictionary, what will be the 50th word?

**3.** What is the number of ways of choosing 4 cards from a pack of 52 playing cards? In how many of there

(i) Four cards one of the same suit

(ii) Four cards belong to four different suits

(iii) Are face cards.

(iv) Two are red cards & two are black cards.

(v) Cards are of the same colour?

**4.** If ${}^nP_r = {}^nP_{r+1}$ and ${}^nc_r = {}^nc_{r-1}$ find the value of n and r.

**5.** Find the value of such n that.

$(i)\ {}^nP_5 = 42\,{}^nP_3,\ n > 4$ $\qquad (ii)\ \frac{{}^nP_4}{{}^{n-1}P_4} = \frac{5}{3},\ n > 4$

# Answer Key:

## MCQ:

**1.** (b) 2880

**2.** (d) x = 12y

**3.** (a) 720

**4.** (a) 588

**5.** (d) 12!/{3! × (4!)³}

**6.** (a) 24

**7.** (b) 1

**8.** (d) 6

**9.** (c) 451

**10.** (c) 5040

## Very Short Answer:

**1.**

$$4!-3! = 4.3!-3!$$
$$=(4-1).3!$$
$$=3.3! = 3\times3\times2\times1$$
$$=18$$

**2.**

$^{n}C_{a} = {}^{n}C_{b} \Rightarrow {}^{n}C_{a} = {}^{n}C_{n-b}$

$a = n - b$

$n = a + b$

**3.**

$0! = 1$

**4.**

First flag can be chosen is 5 ways

Second flag can be chosen is 4 ways

By F.P.C. total number of ways = 5 × 4 = 20

**5.** First letter can be used in 10 ways

Second letter can be used in 9 ways

Third letter can be used in 8 ways

Forth letter can be used in 7 ways

By F.P.C total no. of ways = 10.9.8.7

= 5040

**6.** Total no. of possible out comes = 2 × 2 × 2 = 8

**7.**

$\frac{8!}{6!2!} = \frac{8.7.6!}{6!.2.1}$

$= 4 \times 7 = 28$

**8.** Given

$^{n}C_{8} = {}^{n}C_{2} \Rightarrow {}^{n}C_{n-8} = {}^{n}C_{2}$

$n - 8 = 2$

$n = 10$

$\therefore {}^{n}C_{2} = {}^{10}C_{2} = \frac{\lfloor 10}{\lfloor 10-2 \lfloor 2}$

$= \frac{10.9.\lfloor 8}{\lfloor 8 \times 2.1} = 5 \times 9 = 45$

**9.** No. of ways of selecting 9 balls

$= {}^{6}C_{3} \times {}^{5}C_{3} \times {}^{5}C_{3}$

$= \frac{\lfloor 6}{\lfloor 3 \lfloor 3} \times \frac{\lfloor 5}{\lfloor 2 \lfloor 3} \times \frac{\lfloor 5}{\lfloor 2 \lfloor 3}$

$$= \frac{6.5.4\lfloor 3}{6.\lfloor 3} \times \frac{5.4.\lfloor 3}{2.\lfloor 3} \times \frac{5.4.\lfloor 3}{2.\lfloor 3}$$

$$= 20 \times 10 \times 10 = 2000$$

**10.**

$$5.\,^4p_r = 6.\,^5p_{r-1}$$

$$\Rightarrow 5.\frac{\lfloor 4}{\lfloor 4-r} = 6.\frac{\lfloor 5}{\lfloor 5-r+1}$$

$$\Rightarrow \frac{5.\lfloor 4}{\lfloor (4-r)} = \frac{6.5.\lfloor 4}{\lfloor 6-r}$$

$$\Rightarrow \frac{1}{\lfloor 4-r} = \frac{6}{(6-r)(5-r)\lfloor 4-r}$$

$$\Rightarrow (6-r)(5-r) = 6$$

$$\Rightarrow 30 - 6r - 5r + r^2 = 6$$

$$\Rightarrow r^2 - 11r - 5r + r^2 = 6$$

$$\Rightarrow r^2 - 8r - 3r + 24 = 0$$

$$\Rightarrow r(r-8) - 3(r-8) = 0$$

$$\Rightarrow (r-3)(r-8) = 0$$

$r = 3$ or $r = 8$

$\therefore$ r = 3

$r = 8$ Rejected. Because if we put $r = 8$ the no. in the factorial is –ve.

## Short Answer:

**1.**

Part-I In the word MONDAY there are 6 letters

$\therefore n = 6$

4 letters are used at a time

$\therefore r = 4$

Total number of words = $^nP_r$

$$= {}^6P_4 = \frac{\lfloor 6}{\lfloor 6-4}$$

$$= \frac{\lfloor 6}{\lfloor 2} = \frac{6.5.4.3.\lfloor 2}{\lfloor 2} = 360$$

Part-II All letters are used at a time but first letter is a vowel then OAMNDY

2 vowels can be arranged in 2! Ways

4 consonants can be arranged in 4! Ways

∴ Total number of words = 2! × 4!

$= 2 \times 4.3.2.1 = 48$

**2.** Proof L.H.S.

$${}^{n}C_{r} + {}^{n}C_{r-1} = \frac{\lfloor n}{\lfloor n-r \lfloor r} + \frac{\lfloor n}{\lfloor n-r+1 \lfloor r-1}$$

$$= \frac{\lfloor n}{\lfloor (n-r)\, r \lfloor r-1} + \frac{\lfloor n}{(n-r+1)\lfloor n-r \lfloor r-1}$$

$$= \frac{\lfloor n}{\lfloor n-r \lfloor r-1} \left[\frac{1}{r} + \frac{1}{n-r+1}\right]$$

$$= \frac{\lfloor n}{\lfloor n-r \lfloor r-1} \left[\frac{n-\cancel{r}+1+\cancel{r}}{r(n-r+1)}\right]$$

$$= \frac{\lfloor n (n+1)}{\lfloor n-r\,(n-r+1)\lfloor r-1\, r}$$

$$= \frac{\lfloor n+1}{\lfloor n+1-r \lfloor n-r} = {}^{n+1}C_{r}$$

**3.** No. of black balls =5

No. of red balls = 6

No. of selecting black balls = 2

No. of selecting red balls = 3

Total no. of selection = ${}^{5}C_{2} \times {}^{6}C_{3}$

$$= \frac{\lfloor 5}{\lfloor 5-2 \lfloor 2} \times \frac{\lfloor 6}{\lfloor 6-3 \lfloor 3}$$

$$\frac{5 \times 4 \times 3!}{3! \times 2} \times \frac{6 \times 5 \times 4 \times 3!}{3! \times 3 \times 2} = 200$$

**4.** Let us first seat 0 the 5 girls. This can be done in 5! Ways

X G X G X G X G X G X

There are 6 cross marked placer and the three boys can be seated ${}^{6}P_{3}$ in ways

Hence by multiplication principle

The total number of ways

$$= 5! \times {}^{6}P_{3} = 5! \times \frac{6!}{3!}$$

$= 4 \times 5 \times 2 \times 3 \times 4 \times 5 \times 6$

$= 14400$

**5.** In the INVOLUTE there are 4 vowels, namely I.O.E.U and 4 consonants namely M.V.L and T

The number of ways of selecting 3 vowels

Out of 4 = $^4C_3$ = 4

The number of ways of selecting 2 consonants

Out of 4 = $^4C_2$ = 6

∴ No of combinations of 3 vowels and 2 consonants = 4 × 6 = 24

5 letters 2 vowel and 3 consonants can be arranged in 5! Ways

Therefore required no. of different words = 24 × 5! = 2880

## Long Answer:

**1.**

Number of girls = 4

Number of boys = 7

Number of selection of members = 5

(i) If team has no girl

We select 5 boys

∴ Number of selection of 5 members

$= {}^7C_5 = \frac{\lfloor 7}{\lfloor 5 \lfloor 2} = 21$

(ii) At least one boy and one girl the team consist of

| Boy | Girls |
|---|---|
| 1 | 4 |
| 2 | 3 |
| 3 | 2 |
| 4 | 1 |

The required number of ways

$= {}^7C_1 \times {}^4C_4 + {}^7C_2 \times {}^4C_3 + {}^7C_3 \times {}^4C_2 + {}^7C_4 \times {}^4C_1$

$= 7 + 84 + 210 + 140$

$= 441$

(iii) At least 3 girls

| Girls | Boys |
|---|---|
| 3 | 2 |
| 4 | 1 |

The required number of ways

$= {}^4C_3 \times {}^7C_2 + {}^4C_4 \times {}^7C_1 = 84 + 7$

$= 91$

**2.**

In the word 'AGAIN' there are 5 letters in which 2 letters (A) are repeated

Therefore total no. of words $\frac{5!}{2!} = 60$

If these words are written as in a dictionary the number of words starting with Letter A. AAGIN = 4! = 24

The no. of wards starting with G GAAIN $= \frac{4!}{2!} = 12$

The no. of words starting with I IAAGN = $\frac{4!}{2!} = 12$

Now

Total words = 24 + 12 + 12 = 48

$49^{th}$ words = NAAGI

$50^{th}$ word = NAAIG

**3.** The no. of ways of choosing 4 cards form 52 playing cards.

$${}^{52}C_4 = \frac{52!}{4!48!} = 270725$$

(i) If 4 cards are of the same suit there are 4type of suits. Diamond club, spade and heart 4 cards of each suit can be selected in ${}^{13}C_4$ ways

∴ Required no. of selection = ${}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4$

$= 4 \times {}^{13}C_4 = 2860$

(ii) If 4 cards belong to four different suits then each suit can be selected in ${}^{13}C_1$ ways required no. of selection

$= {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 = 13^4$

(iii) If all 4 cards are face cards. Out of 12 face cards 4 cards can be selected in ${}^{12}C_4$ ways.

∴ required no. of selection ${}^{12}C_4$ $= \frac{\lfloor 12}{\lfloor 8 \lfloor 4} = 495$

(iv) If 2 cards are red and 2 are black then. Out of 26 red card 2 cards can be selected in ways similarly 2 black card can be selected in $^{26}C_2$ ways

$\therefore$ required no. of selection $= {}^{26}C_2 \times {}^{26}C_2$

$= \frac{26!}{2!4!} \times \frac{26!}{2!4!} = (325)^2$

$= 105625$

(v) If 4 cards are of the same colour each colour can be selected in $^{26}C_4$ ways

Then required no. of selection

$= {}^{26}C_4 + {}^{26}C_4 = 2 \times \frac{26!}{4!22!}$

$= 29900$

**4.**

Given that

$^nP_r = {}^nP_{r+1}$

$\Rightarrow \frac{\lfloor n}{\lfloor n-r} = \frac{\lfloor n}{\lfloor n-r-1}$

$\Rightarrow \frac{1}{(n-r)\lfloor n-r-1} = \frac{1}{\lfloor n-r-1}$

$\Rightarrow n-r = 1 \ldots\ldots\ldots (i)$

also $^nC_r = {}^nC_{r-1}$

$\Rightarrow \frac{\lfloor n}{\lfloor n-r \lfloor r} = \frac{\lfloor n}{\lfloor n-r+1 \lfloor r-1}$

$\Rightarrow \frac{1}{\lfloor n-r \; r\lfloor r-1} = \frac{1}{(n-r+1)\lfloor n-r \lfloor r-1}$

$\Rightarrow \frac{1}{r} = \frac{1}{n-r+1}$

$\Rightarrow n-2r = -1 \ldots\ldots\ldots (ii)$

Solving eq $(i)$ and eq $(ii)$ we get $n = 3$ and $r = 2$

**5.**

(i) $^nP_5 = 42\,{}^nP_3$

$\Rightarrow \frac{\lfloor n}{\lfloor n-5} = 42\frac{\lfloor n}{\lfloor n-3}$

$\Rightarrow \frac{1}{\lfloor n-5} = \frac{42}{(n-3)(n-4)\lfloor n-5}$

$\Rightarrow (n-3)(n-4)=42$

$\Rightarrow n^2-4n-3n+12=42$

$\Rightarrow n^2-7n-30=0$

$\Rightarrow n^2-10n+3n-30=0$

$\Rightarrow n(n-10)+3(n-10)=0$

$\Rightarrow (n+3)(n-10)=0$

$n=-3$ or $n=10$

$n=-3$ Is rejected

Because negative factorial is not defined $\therefore n=10$

(ii)

$\frac{{}^{n}P_4}{{}^{n-1}P_4}=\frac{5}{3} \qquad n>4$

$\Rightarrow \frac{\frac{\lfloor n}{\lfloor n-4}}{\frac{\lfloor n-1}{\lfloor n-5}}=\frac{5}{3}$

$\Rightarrow \frac{\lfloor n}{\lfloor n-4}\times\frac{\lfloor n-5}{\lfloor n-1}=\frac{5}{3}$

$\Rightarrow \frac{n\lfloor n-1}{(n-4)\lfloor n-5}\times\frac{\lfloor n-5}{\lfloor n-1}=\frac{5}{3}$

$\Rightarrow 3n=5n-20$

$\Rightarrow -2n=-20 \quad \Rightarrow n=10$